

Statistics:

Poisson Distribution



Before reading, make sure you are familiar with the concept of expected value and variance (see handout available on this).

What is the Poisson Distribution?

The Poisson Distribution is a discrete valued probability distribution that takes positive integer values $0, 1, 2, \dots$. It has one parameter λ which controls the shape of the distribution.

The expected value and variance of the Poisson Distribution are both the same! Furthermore, they both equal the value of λ . That is

$$E(X) = Var(X) = \lambda.$$

The higher the value of λ the higher both the expected value and variance are.

What can it be used for?

The Poisson Distribution is used to model count data. A clever example of this is the number of goals in a soccer match. You might expect a soccer match to have 3 or 4 goals but you are not sure. Therefore you might choose a Poisson distribution with $\lambda = 3.5$ as the distribution of goals.

Car Insurance companies also use a Poisson Distribution to predict how many accidents a driver will have in 1 year. A safe driver may have $\lambda = 0.4$ where as a risky drive may have λ as much as 2 or 3

Calculating Probabilities

Imagine the number of car accidents on the M50, X , can be modelled by a Poisson Distribution with a parameter $\lambda = 5$ per week.

Q: What is the probability that less than 3 accidents occur in the next week

A: To calculate this we need to use the probability mass function of the Poisson Distribution. This is:

$$P(X = x) =$$

For there to be less than 3 accidents, either 0, 1, 2 accidents have to occur.

$$\begin{aligned} P(< 3 \text{ accidents}) &= P(X = 0) + P(X = 1) + P(X = 2) \\ &= \\ &= 0.34 \end{aligned}$$

Important: Check that you are able to use the formula correctly to calculate these values.